Dynamic Pricing Startegies for Discrete Perishable Products

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Dynamic Pricing Strategies for Discrete Perishable Products

Abstract:

The paper is devoted to the development of a dynamic pricing strategy for the sale of products whose value expires with time. Sales of discrete goods to heterogeneous consumers who have at most a unit demand for sold goods is considered. This type of demand structure makes it possible to describe the optimal pricing strategy using a simple system of recursive equations, which allows for an analytical solution in some special cases. For the general case, a numerical algorithm has been developed for finding the optimal solution as a function of time to expiration and remaining stocks of unsold products. The proposed strategy is compared with the fixed-price policy and estimates of economic gain from employing dynamically adjusted prices are obtained. Simple heuristics for sales management under conditions of incomplete information are also proposed.

Keywords: dynamic pricing, price discrimination, dynamic programming.

Track:

1. Introduction

The development of e-commerce has intensified interest in dynamic pricing algorithms for sales management. In a broad sense, dynamic pricing can be defined as a policy of setting flexible prices for goods or services based on changing market conditions. Factors influencing prices may include seasonality, time of the day, unexpected changes in demand, inventory holdings, actions of competitors and so on. Dynamic pricing is a form of price discrimination where different buyers pay different prices for the same product.

It should be noted that dynamic pricing has been a standard trade practice for most of human history. Traditionally, the selling price was determined as a result of negotiations between the seller and the buyer, as is practiced in marketplaces today, especially in eastern countries. Such negotiations are time consuming and require a highly qualified salesperson. With the advent of large retail chains, cash registers, factory packaging, expansion of assortments etc. in the late nineteenth century, this practice became inefficient due to economies of scale and the policy of uniform prices became widespread. This was accompanied by the adoption of legislation against price discrimination in a number of countries, for example, the Robinson-Patman Act of 1936 in the United States (Campagna, 2018).

The development of information technology in general and automated trading systems , in particular has led to the revival of dynamic pricing. This was also facilitated by the wave of market liberalization in the 1980s. In modern sales management systems, the role of the seller is often performed by a computer program which can «memorize» a number of demand-affecting conditions unattainable by a human salesperson. Dynamic pricing strategies have become especially popular when selling airplane seats, where the price of the ticket for the same flight can vary by an order of magnitude depending on the season, day of the week, time before departure, booking conditions and other factors. Other industries where dynamic pricing is widespread include hospitality, entertainment, retail, public transportation, etc.

Thus, the study of the mechanisms of dynamic price adjustments for sales management and their impact on the economic performance of the enterprise is important both from the theoretical and practical perspective.

2. Literature Overview and Research Objectives

Probably the first attempt of analyzing the benefits of dynamic price adjustments for sales management from a mathematical point of view is due to Kincaid and Darling (1963). Lazear (1986) considers the problem of selling an indivisible product to homogeneous consumers with an unknown demand function which the seller can gradually learn by changing prices. It has been shown that the optimal policy for the seller is to slowly lower prices over time, effectively employing a practice known in marketing as the "skimming policy". Papers by Pashigian (1988), Pashigian and Bowen (1991) were devoted to the empirical study of the usage of flexible prices in retailing in the United States.

Perhaps the most influential study on this topic is the work of Galego and van Ryzin (1994). They modeled demand as a continuous Poisson process with an intensity that depends on the selling price. Variational calculus methods were used to find the optimal control policy. The authors obtained an analytical solution for the exponential demand function and established some properties of the optimal pricing policy for a more general case. However, in real commercial practice, a change in prices on a continuous basis is hardly possible. There are many modifications of the Galego-van Ryzin model, which consider restrictions on the set of reasonable prices (Feng and Xiao, 2000), non-stationary demand (Zhao and Zheng, 2000), etc. A detailed review of these studies is contained in Elmaghraby and Keskinocak (2003).

Another important area of research is to consider the possibility of strategic behavior on the part of consumers. This topic is largely motivated by the classic work of Nobel Laureate Ronald Coase (1972), which questioned the effectiveness of the skimming policy. According to Coase, rational consumers will anticipate future price reductions and thus postpone their decision to buy the product, which in turn will force the seller to immediately reduce prices, even in a monopoly case. The role of expectations and forward-looking consumer behavior for sales management was analyzed by Su (2007), Melnikov (2013), Levin, McGill and Nediak (2009), among others.

Many recent studies on this issue are devoted to the use of machine learning algorithms to estimate the intensity of consumer flow and the impact of prices on the probability of purchase (Avramidis, 2020; Wang et al., 2021).

However, many aspects of the overall problem of dynamic pricing remain unresolved. In particular, they include such issues as:

– qualitative characteristics of the properties of pricing policy in sales management over a fixed time interval;

– assessment of the cost-effectiveness of dynamic pricing strategies compared to fixed price strategies;

– simple heuristics for price adjustment rules, which do not require processing enormous amount of data, which might be difficult to obtain.

This paper aims to develop a model of dynamic adaptive pricing for the sale of products whose value expires with time which face a discrete demand, where each individual consumer is interested in consuming at most a single unit of a product. A typical example of such a problem is the above-mentioned sale of airline tickets, when it is necessary to sell a fixed number of seats in the cabin before the departure of the flight. Similar situations arise when selling hotel rooms, tickets to concerts and sporting events, perishable goods, and so on. The simple demand structure allows in some cases to obtain an analytical solution for the optimal pricing policy, as well as to characterize its properties using simple heuristics.

3. The Model

Let us consider a simple single-agent, homogeneous products model. The seller maximizes expected discounted revenue from selling x units of a homogeneous discrete good over a finite time interval. Time is discrete, t=1,...,T. Product is perishable and its value expires at time T. Each period there is a single consumer, indexed by t, who has a reservation price r_t. Further assume that r_t are independent identically distributed random variables drawn from the distribution given by cumulative distribution function $F(r)$ with a finite support; $0 \le r_t \le v$. Consumer arriving at time t buys one unit of a good if $p_t \le r_t$, where p_t is the price charged by the seller. Thus, the probability of sale in period t equals $G(p_t)=1-F(p_t)$. Otherwise, consumer buys nothing and the process restarts at the next period with the same inventory level.

Let x_t denote remaining stock at the beginning of period t and assume for simplicity zero storage costs.

The seller's problem is to maximize over a sequence of p_t

$$W(x,t) = E\left[\sum_{t=1}^{T} \beta^t I\{x_t > 0\}(p_t I\{p_t \le r_t\}) \mid x_1 = x\right] \qquad (1)$$

subject to

$$x_{t+1} = x_t - I\{p_t \le r_t\}\ , \qquad (2)$$

where $E[\cdot]$ is the (conditional) expectation operator and $I\{\cdot\}$ is the indicator function.

It is easy to cast this optimal control problem into the standard dynamic programming framework (Stokey and Lucas, 1989; Judd, 1998). To this end, define V_t^x and p_t^x to be the expected discounted revenue and the price as of time *t*, correspondingly, when the current stock equals *x*. Dynamics of the value function V_t^x is given by

$$V_t^x = \max_{p_t^x} \left[G(p_t^x)[p_t^x + \beta V_{t+1}^{x-1}] + F(p_t^x)\beta V_{t+1}^x \right] \tag{3}$$

subject to the boundary conditions

$$V_{T+1}^x = 0 \ \forall x \geq 0; \ V_t^0 = 0 \ \forall t = 1, \ldots, T \ . \tag{4}$$

First order optimality conditions for the above problem are

$$p_t^x - H(p_t^x) = \beta(V_{t+1}^x - V_{t+1}^{x-1}) \ , \tag{5}$$

where p_t^x is an inverse of the hazard rate,

$$H(p_t^x) = \frac{1 - F(p_t^x)}{f(p_t^x)} \ . \tag{6}$$

Substituting (5) into (3) we obtain

$$V_t^x = \beta V_{t+1}^x + G(p_t^x)H(p_t^x) \ . \tag{7}$$

Numerically, above problem can be solved by backward induction. From (5) using boundaries it immediately follows that

$$p_T^x - H(p_T^x) = 0 \ \forall x \geq 0 \ , \tag{8}$$

hence last period price is independent of *x*. Then from (3) we obtain V_t^x, solve for p_{t-1}^x using (5) and so on.

As an example, consider the special case with uniform *U*(0,1) distribution of reservation prices. Then *F*(*p*)=*p*; *f*(*p*)=1; *H*(*p*)=1−*p*. Using (8), we obtain

$$p_T^x = \frac{1}{2}; V_T^x = \frac{1}{4} \forall x \geq 0 \ . \tag{9}$$

Substituting (9) into (5) and (7) for periods *T*−1, *T*−2 and so on, we arrive to the following sequence of optimal prices:

$$\begin{aligned} p_t^1 &= \frac{1 + \beta(p_{t+1}^1)^2}{2}; \\ p_t^x &= \frac{1 + \beta\left[(p_{t+1}^x)^2 - (p_{t+1}^{x-1})^2\right]}{2}; \\ V_t^x &= 1 + \left(p_t^x\right)^2 + 2\sum_{i=1}^{x-1} p_t^i - x. \end{aligned} \tag{10}$$

The p_t^1 price path corresponds to the situation where the seller has a single indivisible unit of a product to sell. This might be of interest by itself for such instances as selling real estate, art objects, and other unique products. The resulting trajectory is shown on the Fig. 1.

It is easy to see that p_t^1 is a decreasing function of time. As time horizon increases, p_{T-t}^1 converges to $\frac{1 - \sqrt{1 - \beta}}{\beta}$, which is an increasing function of *β*. With no discounting p_{T-t}^1 converges to one, which is the upper bound for consumer valuation.

Figure 1. Optimal prices for a single product as a function of time (r_t~U(0,1), β=1)

For arbitrary stock levels, it is straightforward to show that inventory-dependent optimal prices p_t^x defined by (10) are weakly decreasing in x. Fig. 2 shows optimal price sequences for x=5 and T=30.

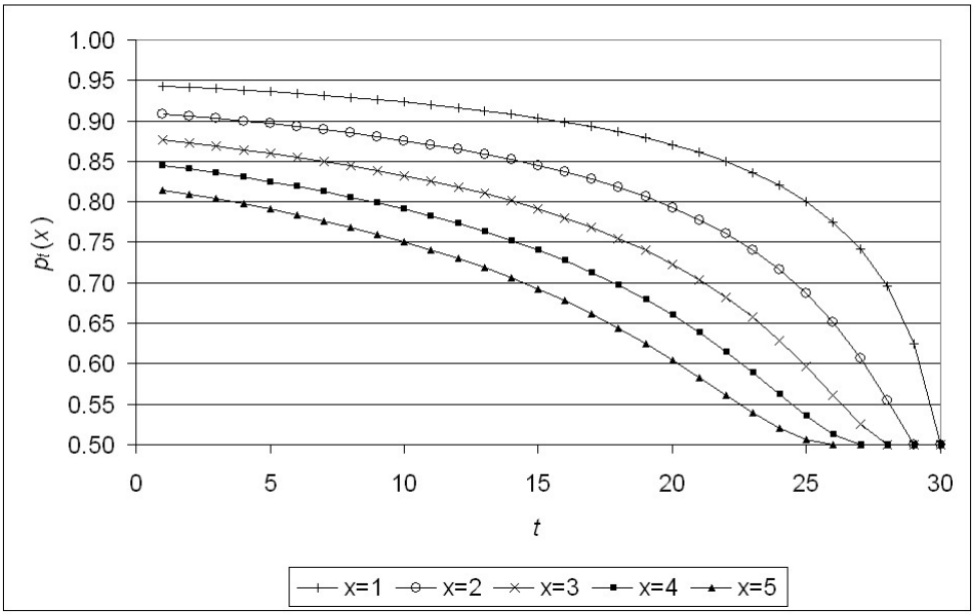

Figure 2. Optimal prices as a function of time and remaining stock levels (r_t~U(0,1), β=1)

As can be seen from Fig. 2, optimal price policy p_t^x is weakly decreasing both in time t and in remaining stock levels x. This remains true for the general case defined by equations of (5)-(7). It is a combination of these properties that produces rich dynamics of observed price paths. An independent observer who has no information on x sees only one of the values of p_t^x as the product selling price at any given time. With stable stock levels, prices will be gradually lowered over time. However, when sale happens, inventory levels drop, and the optimal price switches from p_t^x to p_t^{x-1}, which may cause the observed product price to rise sharply. As the timing of a successful sale is random, so is the observed price path.

The mechanism of this process is illustrated on Fig. 3, where price fluctuations (black line) are superimposed on possible inventory movements (blue line) obtained as a result of computer simulation.

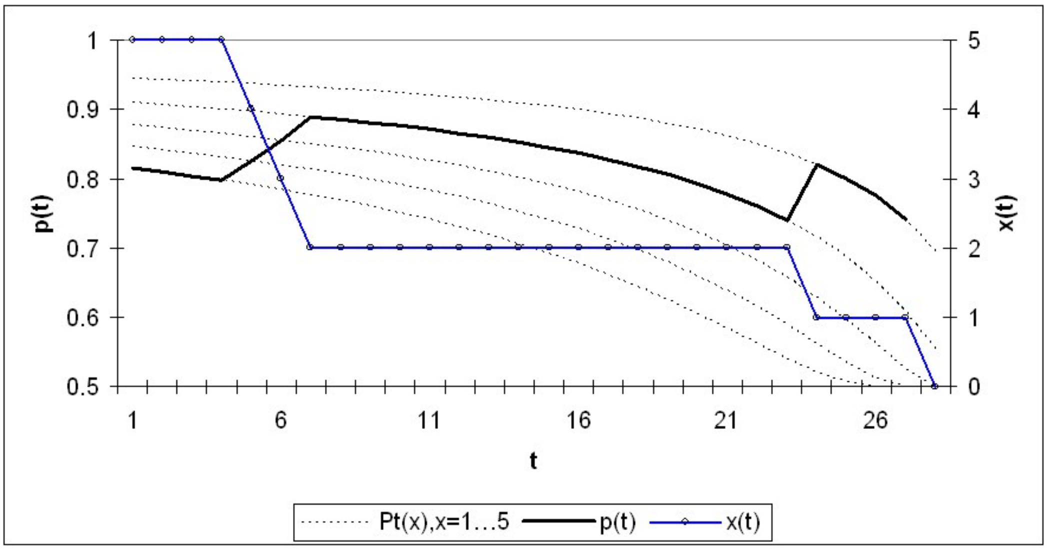

Fig. 3. Price adjustment mechanism

Fig. 4 shows results of several computer simulations that illustrate possible price dynamics that may arise in the above model. As evident from these graphs, prices may decline over time just as easily as they can rise up.

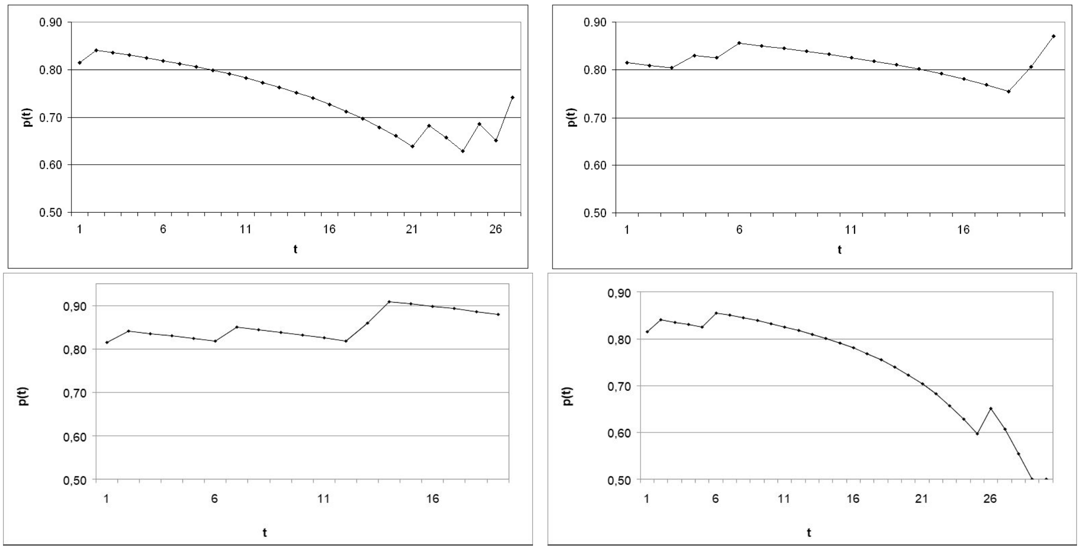

Figure 4. Some simulated price paths

4. Benchmarking

To assess efficiency of the proposed strategy, let us compare seller's expected revenue at fixed prices and under dynamic price adjustment according to the proposed scheme. To keep things simple, consider the case of selling a single product discussed above.

At the fixed price p, the probability that a product will not be sold during T periods of time equals $F(p)^T$. Hence, the probability of a successful sale equals $1–F(p)^T$, and the seller's expected revenue is given by

$$W(p) = p\times(1–F(p)^t). \quad (11)$$

The optimal fixed price can be found by maximizing above expression over p. For instance, if r_t~$U(0,1)$ as in the example illustrated by Fig. 1, it is straightforward to show that the optimal fixed price equals $(1+T)^{1/T}$. Expected revenue can be obtained by plugging this formula into (11).

Fig. 5 compares expected seller's revenues under the optimal fixed price policy (dotted line) and under dynamic price adjustment (solid line). Relative efficiency of employing the second strategy is also shown.

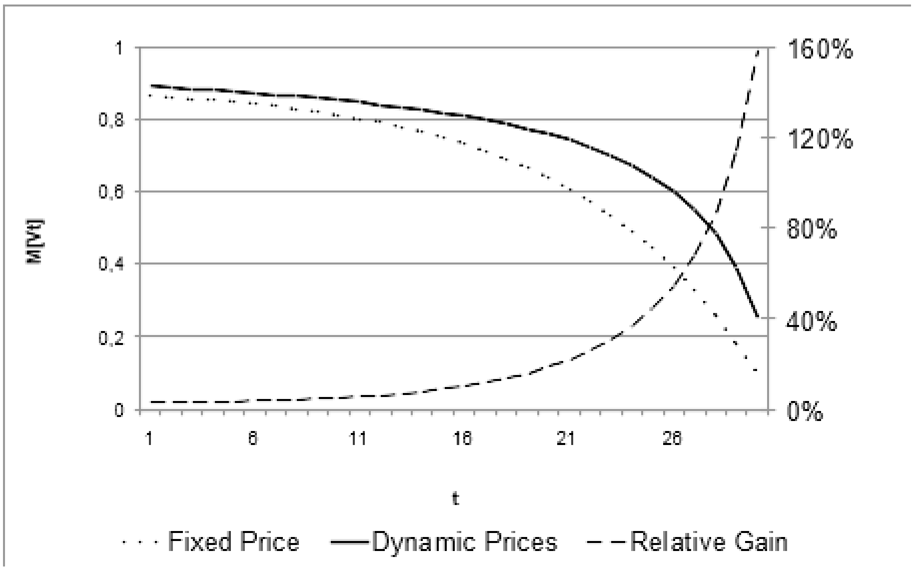

Figure 5. Comparison of the seller's expected revenue under fixed and flexible prices

As can be seen from the graph, with a fairly long realization time horizon the benefits of using dynamic pricing scheme are relatively small. However, they grow rapidly when there is little time left before the expiration of the product value. According to calculations, in relative terms the gain from dynamic pricing ranges from 3% to 159%.

5. Implementation and Simple Pricing Heuristics

The assumptions made about the structure of demand for seller's products in conjunction with the solution of the system of equations (5)−(7) jointly create a Markov process for the evolution of inventories x_t during the sales period. In turn, they define the distribution of the observed product prices p_t. The probabilities of certain states of the x_t process are determined by recursive equations

$$\begin{aligned}&\pi_t(x_t = x) = \pi_{t-1}(x_{t-1} = x)(1 - Q(p_{t-1}^x)) + \pi_{t-1}(x_{t-1} = x+1)Q(p_{t-1}^x)\ \forall x = 1,2,...;\\&\pi_t(x_t = 0) = \pi_{t-1}(x_{t-1} = 0) + \pi_{t-1}(x_{t-1} = 1)Q(p_{t-1}^1).\end{aligned} \quad (12)$$

where the initial probability distribution under $x_0 = X$ is given by:

$$\pi_0(x_0 = X) = 1;\ \pi_0(x_0 = x) = 0\ \forall x = 0,1,...,X-1. \quad (13)$$

Formulae (12)−(13) allow to determine the expected values of prices, sales, inventories, seller's revenue and any other numerical characteristics of random processes x_t, p_t and their derivatives for each time period .

Below are some interesting properties of optimal pricing strategies obtained by numerical experiments for the case of standard uniform distribution of consumer reservation prices considered above.

Fig. 6 shows the conditional expected value of the product price as a function of time, provided that the batch of products has not yet been sold, i.e. $E[p_t \mid x_t > 0]$. As can be seen from the graph, for a long period of time product prices remain approximately constant. The rapid decline in product prices occurs only towards the end of sale period, if the batch of products failed to sell out ahead of schedule.

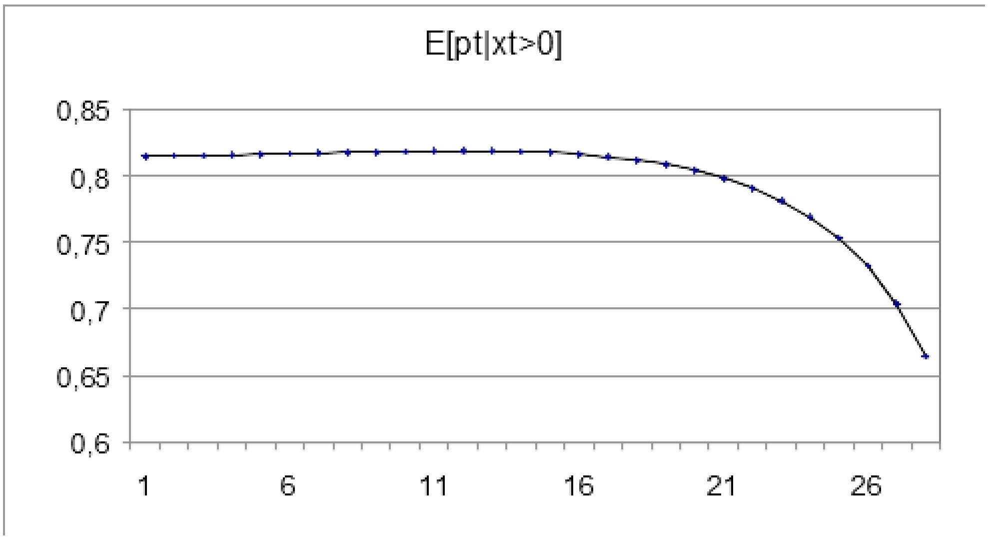

Figure 6. Expected product price as a function of time

Fig. 7 shows the expected level of unsold product stocks as a function of time, i.e. $E[x_t]$ (dashed line). Apparently, the pricing policy calculated on the basis of the above model results in a constant expected sales intensity.

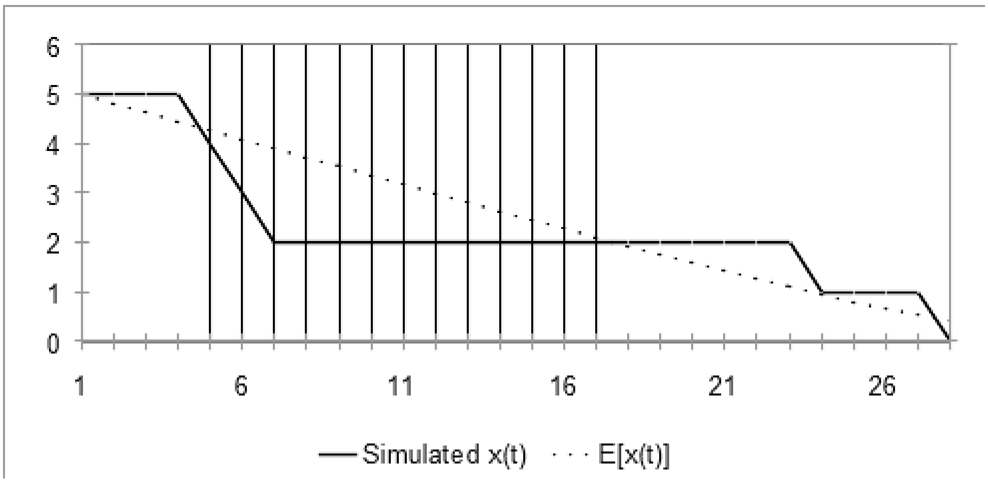

Figure 7. Expected value and a simulated path of the x_t random process

Fig. 7 also provides an intuitive explanation for the price adjustment mechanism in the described model. To this end, a graph of the expected inventory levels x_t is superimposed over a particular realization of this process obtained in the simulation experiment shown in Fig. 2. Vertical lines indicate those periods of time when prices in the simulation experiment exceeded their expected level (shown in Fig. 6). They coincide with those periods of time when unsold inventories in the experiment were below their expected level (dashed line in

Fig. 7). If we interpret the graph of $E[x_t]$ as a target sales plan, the workings of the mechanism of price adjustment in the above model can be described by the following simple heuristic. If sales proceed faster than planned, the price of products rises. On the other hand, if the sales process slows down compared to the target, prices fall. This rather intuitive rule can form the basis for the algorithm of dynamic price adjustment in cases where available data are insufficient to implement the above model.

6. Conclusions and Directions for Further Research

The model of dynamic price adjustment for managing sales of discrete perishable products developed above allows to determine optimal prices as a function of time before the expiration date and unsold inventory stocks. Computer simulations show the high efficiency of the proposed mechanism in comparison to the fixed price strategy. Based on the established properties of the optimal pricing strategy, it is possible to develop fairly simple heuristic mechanisms for flexible price regulation.

Practical implementation of the above model requires estimates of the intensity of consumer flow and theirb price sensitivity, which may not be readily available. Thus, a perspective direction of research would be to integrate the above model with machine learning algorithms for analytical processing of the sales data.

Another interesting expansion of the model would be to consider the above problem as a repeating process. From this perspective, pricing decisions are closely related to classical inventory management problems such as determining the optimal order size, scheduling of inventory replenishing and so on. Studying how dynamic pricing affects those decisions may be of substantial theoretical and practical interest.

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